The Question of Low-Lying Intruder States in ^8Be and Neighboring Nuclei

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The presence of not yet detected intruder states in ^8Be e.g. a $J=2^+$ intruder at 9 MeV excitation would affect the shape of the $\beta^{\mp}$-delayed alpha spectra of ^8Li and ^8B. In order to test the plausibility of this assumption, shell model calculations with up to $4\hbar\omega$ excitations in ^8Be (and up to $2\hbar\omega$ excitations in ^{10}Be) were performed. With the above restrictions on the model spaces, the calculations did not yield any low-lying intruder state in ^8Be. Another approach -the simple deformed oscillator model with self-consistent frequencies and volume conservation gives an intruder state in ^8Be which is lower in energy than the above shell model results, but its energy is still considerably higher than 9 MeV.

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I. INTRODUCTION AND MOTIVATION

In an R matrix analysis of the $\beta^{\mp}$-delayed alpha spectra from the decay of ^8Li and ^8B as measured by Wilkinson and Alburger [1], Warburton [2] made the following statement in the abstract: "It is found that satisfactory fits are obtained without introducing intruder states below 26-MeV excitations". However, Barker has questioned this [3,4] by looking at the systematics of intruder states in neighboring nuclei. He noted that the excitation energies of 0_2^+ states in ^{16}O, ^{12}C and ^{10}Be were respectively 6.05 MeV, 7.65 MeV and 6.18 MeV. Why should there not then be an intruder state in ^8Be around that energy?

In recent works [5,6] the current authors and S. S. Sharma allowed up to $2\hbar\omega$ excitations in ^8Be and in ^{10}Be, and indeed $2p-2h$ intruder states were studied with some care in ^{10}Be. Using a simple quadrupole-quadrupole interaction $-\chi Q\cdot Q$ with χ=0.3615 MeV/fm^4 for ^{10}Be and $\hbar\omega = 45/A^{1/3} - 25/A^{2/3}$. We found a $J=0^+$ intruder state at 9.7 MeV excitation energy. This is higher than the experimental value of 6.18 MeV, but it is in the ballpark. However, there are other $J=0^+$ excited states below the intruder state found in the calculation.

In a $0p$-shell calculation with the interaction $-\chi Q\cdot Q$, using a combination of the Wigner Supermultiplet theory [7] characterized by the quantum numbers $[f_1 f_2 f_3]$ and Elliott's $SU(3)$ formula [8], one can obtain the following expression giving the energies of the various states:

$$E(\lambda\ \mu) = \bar{\chi}\left[-4(\lambda^2+\mu^2+\lambda\mu+3(\lambda+\mu))+3L(L+1)\right] \tag{1}$$

where

$$\lambda = f_1 - f_2, \quad \mu = f_2 - f_3 \tag{2}$$

and

$$\bar{\chi} = \chi\frac{5b^4}{32\pi} \quad (b^2 = \frac{\hbar}{m\omega}) \tag{3}$$

The two $J=0^+$ states lying below the calculated intruder state in ^{10}Be, at least in the calculation, correspond to two degenerate configurations [411] and [330]. Both of these have configurations $L=1$ $S=1$ from which one can form the triplet configurations $J=0^+,\ 1^+,\ 2^+$. Hence, besides the intruder state, we have the above two $J=0^+$ states as candidates for the experimental 0_2^+ state at 6.18 MeV.

As noted in the previous work [5] if, in the $0p$-shell model space we fit χ to get the energy of the lowest 2^+ state in ^{10}Be to be at the experimental value of 3.368 MeV ($18\bar{\chi}$), then the two sets of triplets are at an excitation energy of 30 $\bar{\chi}$ which equals 5.61 MeV -not far from the experimental value. There is however a problem -in a $0p$-space calculation with $Q\cdot Q$, the lowest 2^+ state is two-fold degenerate, corresponding to $J=2^+$ $K=0$ and $J=2^+$ $K=2$.

So it is by no means clear if the 0^+ state in ^{10}Be at 6.18 MeV is an intruder state. We will discuss this more in a later section. It should be noted that in the previously mentioned calculation [6], the energy of the intruder state is very sensitive to the value of χ, the strength of the $Q \cdot Q$ interaction. The energy of this intruder state drops down rapidly and nearly linearly with increasing χ.

Because of uncertainties due to the truncations in the shell model calculations, an alternate approach is also considered. This is the deformed oscillator model with volume conservation and self-consistent frequencies.

II. RESULTS OF THE SHELL MODEL DIAGONALIZATIONS

In Tables I, II and III we give results for the energies of $J = 0^+$ and 2^+ states in ^8Be, in which up to $4\hbar\omega$ excitations are allowed relative to the basic configurations $(0s)^4(0p)^4$. The different tables correspond to different interactions as follows:
(a) Quadrupole-Quadrupole: $V = -\chi Q \cdot Q$ with $\chi = 0.3467\ MeV/fm^4$.
(b) $V = -\chi Q \cdot Q\ +\ xV_{s.o.}$ (χ as above and $x = 1$).
(c) $V = V_c + xV_{s.o.} + yV_t$ ($x = 1,\ y = 1$).

Case (c) above consists of a simplified realistic interaction constructed by Zheng and Zamick [9]. They took a combination of a central V_c, a spin-orbit $V_{s.o.}$ and a tensor interaction V_t and fitted the parameters to the realistic Bonn A bare G matrix elements [10]. To study the effects of varying the spin-orbit and tensor interactions they multiplied these by factors x and y, respectively. For $x = 1,\ y = 1$ one gets the best fit to the Bonn A matrix elements and this choice is used in this work. This has been discussed extensively in previous references [5,9,11].

It should be noted that in all our shell model matrix diagonalizations the effects of spurious center of mass motion are removed. In the OXBASH program used here [12], this is done by using the Gloeckner-Lawson method which pushes the spurious states to a very high energy. For more details see Refs. [11,13].

In Tables IV, V and VI we present results for isospin one $J = 0^+$ and 2^+ states in ^{10}Be in which up to $2\hbar\omega$ excitations have been included. We have the same three interactions as above but with $\chi = 0.3615\ MeV/fm^4$ in (a) and (b).

In all the tables we give the excitation energies of the $J = 0^+$ and 2^+ states and the percent probability that there are no excitations beyond the basic configuration ($0\hbar\omega$) and the percentage of $2\hbar\omega$ excitations (as well as $4\hbar\omega$ excitations for ^8Be).

Note that for interaction (a) the respective percentages for the ground state of ^8Be (see Table I) are 62.8%, 25.7% and 11.5%: there is considerable mixing. Thus we should not forget, when we discuss the question "where are the intruder states?", that there is considerable admixing of $2\hbar\omega$ and $4\hbar\omega$ excitations *in the ground state*. Note that the ground state configuration does not change very much for the three interactions that are considered here. For example, as seen in Table III, the corresponding percentages for the (x, y) interaction are 62.2%, 26.2% and 11.6%.

By looking at these tables, it is not too difficult to see at what energies the intruder states set in. One sees a sharp drop in the $0\hbar\omega$ occupancy. For example in Table I, whereas the $0\hbar\omega$ percentage for the 18.7 MeV and 20.2 MeV states are respectively 93.9% and 94.6%, for the next state at 26.5 MeV the percentage drops to 29.4% -also the next four states listed have very low $0\hbar\omega$ percentages and are therefore intruders.

The terminology *intruder state* is somewhat arbitrary. It is used by experimentalists to refer to certain low-lying states with certain properties. In shell model calculations it is generally used for states whose main components are outside the model space composed of one major shell N (the valence shell). Following this criterion in our theoretical calculations we define an intruder state as one for which the $0\hbar\omega$ percentage is less than 50%. By this criterion, and for the three interactions discussed here, the lowest $J = 0^+$ intruder states in ^8Be are at 26.23 MeV, 26.5 MeV and 28.7 MeV (see Tables I,II, and III). The $J = 2^+$ intruder states are at 27.15 MeV, 27.5 MeV and 33.7 MeV. Note that up to $4\hbar\omega$ excitations were allowed in these calculations. These energies are very high and would argue against the suggestion by Barker that there are low-lying intruder states in ^8Be.

What about ^{10}Be? Remember that in this nucleus we only include up to $2\hbar\omega$ excitations. For the three interactions considered, the lowest $J = 0^+\ T = 1$ intruder states are at 9.7 MeV, 11.4 MeV and 31.0 MeV. The 'anomalous' behavior for the last value (31.0 MeV for the (x, y) interaction) will be discussed in a later section.

Note that when a spin-orbit is added to $Q \cdot Q$, the energy of the intruder state goes up *e.g.* 11.4 MeV *vs* 9.7 MeV. The lowest-lying $J = 2^+\ T = 1$ intruder states are at 11.9 MeV, 13.8 MeV and 33.4 MeV. The energy of the non-intruder ($L = 1\ S = 1$) $J = 0^+,\ 1^+,\ 2^+$ triplet also goes up as can be seen from Tables IV and V.

For the two $Q \cdot Q$ interactions, the energies of the intruder states in ^{10}Be are much lower than in ^8Be. This conclusion still holds if we were to use ^8Be energies calculated in (0+2)$\hbar\omega$ configuration space -see Table VII. This

would indicate that even if we do find low-lying intruder states in ^{10}Be, such a finding in itself is not proof that they are also present in ^{8}Be. Indeed, our calculations would dispute this claim.

III. $(0+2)\hbar\omega$ VS $(0+2+4)\hbar\omega$ CALCULATIONS FOR ^{8}BE

In Table VII we show the results for the energy of the first intruder state in ^{8}Be in calculations in which only up to $2\hbar\omega$ excitations are included and compare them with the corresponding results for up to $4\hbar\omega$. For interactions (a) and (b), the value of χ was changed to 0.4033 MeV/fm^4 in order that the energy of the 2_1^+ state come close to experiment. In more detail, we have to rescale χ depending on the model space in order to get the 2_1^+ state at the right energy. In general, the more $np-nh$ configurations we include the smaller χ is.

We see that in the larger-space calculation $(0+2+4)\hbar\omega$, the energies of the lowest intruder states in most cases come down about 5 MeV relative to the $(0+2)\hbar\omega$ calculation. The excitation energies are still quite high, however, all being above 25 MeV. One possible reason for the difference between the results of the two calculations is that in the $(0+2)\hbar\omega$ calculation there is level repulsion between the $0\hbar\omega$ and the $2\hbar\omega$ configurations, and that the $4\hbar\omega$ configurations are needed to push the $2\hbar\omega$ states back down.

IV. THE FIRST EXCITED $J=0^+$ STATE OF ^{10}BE

Is the first excited $J=0^+$ state in ^{10}Be an intruder state or is it dominantly of the $(0s)^4(0p)^6$ configuration? Experimentally, very few states have been identified in ^{10}Be. The known positive-parity states are as follows [14]:

J^π	$E_x(MeV)$
0_1^+	0.000
2_1^+	3.368
2_2^+	5.959
0^+	6.179
2^+	7.542
(2^+)	9.400

In the $(0s)^4(0p)^6$ calculation with a $Q{\cdot}Q$ interaction, the lowest 2^+ state at $18\bar{\chi}$ is doubly degenerate and corresponds to $K=0$ and $K=2$ members of the [42] configuration. There are two degenerate ($L=1$ $S=1$) configurations at $30\bar{\chi}$ with supermultiplet configurations [330] and [411]. From $L=1$ $S=1$ one can form a triplet of states with $J=0^+,\ 1^+,\ 2^+$. If we choose $\bar{\chi}$ by getting the 2_1^+ state correct at 3.368 MeV, then the two $L=1$ $S=1$ triplets would be at $30/18\times 3.36$ $MeV = 5.61$ MeV. However, there should be a *triplet* of states. In more detailed calculations, as the spin-orbit interaction is added to the $Q\cdot Q$ interaction, the triplet degeneracy gets removed with the ordering $E_{2^+} < E_{1^+} < E_{0^+}$. As seen in Table IV, the $J=0^+$ and 2^+ states of ^{10}Be at 3.7 MeV and 7.3 MeV are degenerate with a pure $Q\cdot Q$ interaction. This is also true for $J=1^+$. In Table V, however, when the spin-orbit interaction is added to $Q\cdot Q$, we find that whereas the 0_2^+ is at 8.0 MeV, the 2_3^+ state is at 6.8 MeV.

Hence if the 0^+ state at 6.179 MeV were dominantly an $L=1$ $S=1$ non-intruder state, one would expect a $J=1^+$ and a $J=2^+$ state at lower energies. Thus far no $J=1^+$ level has been seen in ^{10}Be but this is undoubtedly due to the lack of experimental research on this target. Now there *is* a lower 2^+ state at 5.959 MeV. This could be a member of the $L=1$ $S=1$ triplet *or* it could be the $K=2$ state of the [42] configuration.

Hence, one possible scenario is that indeed the 2_2^+ state is dominantly of the [42] configuration and the $J=0_2^+$ state is a singlet. This would support the idea that the $J=0_2^+$ state is an intruder state. The second scenario has the $J=2_2^+$ state being dominantly an $L=1$ $S=1$ state for which the $J=1^+$ member has somehow not been found. This would be in support of the idea that the 0_2^+ state is *not* an intruder state.

Let us look in detail at Tables IV, V and VI which show where the energies of the intruder states are in a $(0{+}2)\hbar\omega$ calculation. For the $Q\cdot Q$ interaction (with $\chi=0.3615$ MeV/fm^4), the lowest $J=0^+$ intruder state is at 9.7 MeV and the lowest $J=2^+$ intruder state is at 11.9 MeV. These energies are *much lower* than the corresponding intruder state energies for ^{8}Be. This in itself is enough to tell us that the presence of a low-energy intruder state in ^{10}Be does not imply that there should be a low energy intruder state in ^{8}Be. Note that the intruder states in this model space and with this interaction have 100% '$2\hbar\omega$' configurations. This has been noted and discussed in [6] and is due to the fact that the $Q\cdot Q$ interaction cannot excite two nucleons from the N shell to the $N\pm 1$ shell.

Still, in Table IV, there are two $J=0^+$ states (below the intruder state) at 3.7 MeV and 7.3 MeV. Even in this large-space calculation, these are members of degenerate $L=1$ $S=1$ triplets $J=0^+,\ 1^+,\ 2^+$. Indeed, if we look down the table, we see the 3.7 MeV and 7.3 MeV values in the $J=2^+$ column.

In Table V, when we add the spin-orbit interaction to $Q \cdot Q$, the energies of the 0_2^+ and 0_3^+ states go up, but so does the energy of the $J = 0_4^+$ intruder state. The energies of the 0_2^+, 0_3^+ and 0_4^+ (intruder) states in Table IV are 3.7, 7.3 and 9.7 MeV; in Table V, with the added spin-orbit interaction they are 8.0, 9.6 and 11.4 MeV.

In Table VI we show results of an up-to-$2\hbar\omega$ calculation with the realistic interaction. Here, we see a drastically different behavior for the intruder state energy in ^{10}Be. The lowest $J = 0^+$ intruder state is at 31.0 MeV, and the lowest $J = 2^+$ intruder state is at 33.4 MeV (recall our operational definition -an intruder state has less than 50% of the $0\hbar\omega$ configuration). For the $Q \cdot Q$ interaction, in contrast, the intruder state was at a much lower energy. A possible explanation is that for the (x, y) interaction, unlike $Q \cdot Q$, one *does have* large off-diagonal matrix elements in which two nucleons are excited from N to $N \pm 1$ *e.g.* from $0p$ to $1s - 0d$. This will cause a large level repulsion between the $0\hbar\omega$ and the $2\hbar\omega$ configurations and drive them far apart. Presumably, if we included $4\hbar\omega$ configurations into the model space, they would push the $2\hbar\omega$ configurations back down to near their unperturbed positions.

Thus, the problem is rather difficult to sort out theoretically, so we can at best suggest that more experiments be done on ^{10}Be. For example, the $B(E2)$ to the 2_2^+ state would be useful. There should be a much larger B(E2) to the $L = 2$ $K = 2$ member of a [42] configuration than to an ($L = 1$ $S = 1$) state. We also predict a substantial $B(M1) \uparrow$ to the first $J = 1^+$ $T = 1$ state in ^{10}Be. Whereas with a pure $Q \cdot Q$ interaction the $B(M1)$ to this state would be zero, the presence of a spin-orbit interaction will 'light up' the 1_1^+ state in ^{10}Be. The $J = 1^+$ should be seen.

V. THE DEFORMED OSCILLATOR MODEL WITH VOLUME CONSERVATION AND SELF-CONSISTENT FREQUENCIES

As an alternative to the shell model approach for finding the energies of intruder states, we consider the deformed oscillator model of Bohr and Mottelson [15]. The Hamiltonian is a sum of one-body terms, one of which is

$$H = -\frac{\hbar^2}{2m}\nabla^2 + \frac{m}{2}(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2) \tag{4}$$

Furthermore, we assume volume conservation:

$$\omega_x \omega_y \omega_z = \omega_0^3 \equiv constant \tag{5}$$

The intrinsic energy is given by

$$E_{int} = \Sigma_x \hbar\omega_x + \Sigma_y \hbar\omega_y + \Sigma_z \hbar\omega_z \tag{6}$$

where $\Sigma_x = \sum(N_x + 1/2)$ where N_x is the number of quanta in the $x-$direction.

The self-consistency condition is

$$\Sigma_x \omega_x = \Sigma_y \omega_y = \Sigma_z \omega_z \tag{7}$$

This can be obtained by minimizing the kinetic energy -indeed for a two-body delta interaction the potential energy depends only on ω_0 and not on the deformation. With this condition, the energy is given by $E_{int} = 3\Sigma_z \hbar\omega_z = 3\hbar\omega_0 \left(\Sigma_x \Sigma_y \Sigma_z\right)^{1/3}$.

For a simple estimate, we take $\hbar\omega_0 = 45A^{-1/3} - 25A^{-2/3}$. This model has been previously applied by L. Zamick *et. al.* [16].

The calculations for the intrinsic states are remarkably simple. One just has to evaluate Σ_x, Σ_y and Σ_z for the ground state and the intruder states. The single-particle states are classified as (N_x, N_y, N_z). The relevant ones for this calculation are (0,0,0), (0,0,1), (1,0,0), (0,1,0) and (0,0,2). For example, for the ground state of ^8Be, the states (0,0,0) and (0,0,1) are occupied so that one has:

$\Sigma_x = 4 \times 1/2 + 4 \times 1/2 = 4$
$\Sigma_y = 4 \times 1/2 + 4 \times 1/2 = 4$
$\Sigma_z = 4 \times 1/2 + 4 \times 3/2 = 8$

For the $2p - 2h$ intruder states, there are four nucleons in (0,0,0), two in (0,0,1) and two in (0,0,2). Hence,

$\Sigma_x = \Sigma_y = 8 \times 1/2 = 4$
$\Sigma_z = 4 \times 1/2 + 2 \times 3/2 + 2 \times 5/2 = 10$

For the ground state, the volume conservation condition ($\omega_x \omega_y \omega_z = \omega_0^3$) becomes:

$8/4 \times 8/4 \times \omega_z^3 = \omega_0^3$ and

$$\frac{\omega_z}{\omega_0} = 0.62996$$

The intrinsic state energy is then $E = 3 \times 8 \times 0.62996\hbar\omega_0 = 15.1990\hbar\omega_0$. The calculations for other states and other nuclei are carried out in the same way.

In order to compare our results with experiment we must obtain the energies of the $J = 0^+$ and $J = 2^+$ states. The 0^+ and 2^+ energies are computed as follows.

In the **axial case**, for a given intrinsic configuration,

$$E_{2^+} - E_{0^+} = \frac{3}{\mathcal{I}} \,, \tag{8}$$

$$E_{0^+} = E_{int} - \Delta E_R \tag{9}$$

where the zero point energy [17]

$$\Delta E_R = \frac{\langle J^2 \rangle}{2\mathcal{I}} \tag{10}$$

with $\langle J^2 \rangle$ the expectation value of the angular momentum squared

$$\langle J^2 \rangle = \langle J_\perp^2 \rangle = \langle J_x^2 \rangle + \langle J_y^2 \rangle = 2 \langle J_x^2 \rangle \tag{11}$$

and $\mathcal{I}$ the cranking moment of inertia for the corresponding configuration, i.e.,

$$\langle J_x^2 \rangle = \sum_{ph} \left| \langle p \left| j_x \right| h \rangle \right|^2 \tag{12}$$

$$\mathcal{I} = \mathcal{I}_x = 2 \sum_{ph} \frac{\left| \langle p \left| j_x \right| h \rangle \right|^2}{\epsilon_p - \epsilon_h} \tag{13}$$

with h and p the occupied and unoccupied states, respectively, in the configuration at hand.

In the **triaxial case** (see for instance [18]) there are two 2^+ states

$$E_{2^+} - E_{0^+} = \left(\frac{1}{\mathcal{I}_x} + \frac{1}{\mathcal{I}_y} + \frac{1}{\mathcal{I}_z} \right) \left\{ 1 \mp \left[1 - \frac{3}{8} \frac{2\mathcal{I}_x\mathcal{I}_z\mathcal{I}_y \left(4\mathcal{I}_x + 4\mathcal{I}_y + 3\mathcal{I}_z\right) + \mathcal{I}_z^2 \left(\mathcal{I}_x^2 + \mathcal{I}_y^2\right)}{\left(\mathcal{I}_x\mathcal{I}_z + \mathcal{I}_y\mathcal{I}_z + \mathcal{I}_x\mathcal{I}_y\right)^2} \right] \right\}^{1/2} \tag{14}$$

The lowest of these 2^+ states is given in the table for the case of the triaxial configuration in ^{10}Be and can be also obtained from the simpler equation

$$E_{2^+} - E_{0^+} \simeq \frac{3}{2} \left(\frac{1}{\mathcal{I}_x} + \frac{1}{\mathcal{I}_y} \right) \tag{15}$$

The zero point energy in the triaxial case is obtained as

$$\Delta E_R = \left(\frac{\langle J_x^2 \rangle}{\mathcal{I}_x} + \frac{\langle J_y^2 \rangle}{\mathcal{I}_y} + \frac{\langle J_z^2 \rangle}{\mathcal{I}_z} \right) \tag{16}$$

VI. DISCUSSION OF RESULTS

A. Experimental situation

We present results for ^8Be, ^{10}Be, and ^{12}C. The latter nucleus is included because there is a known $J = 0^+$ intruder state at 7.654 MeV, generally considered to be a $4p - 4h$ state. In ^{10}Be there is a $J = 0^+$ excited state at 6.11 MeV, which may well be a $2p - 2h$ intruder state. However ^{10}Be is a remarkably understudied nucleus and it would be nice to have more experimental work to confirm (or deny) this. Although we will not include calculations for ^{11}Be here, it should be noted that for this nucleus there is an inversion with a $J = 1/2^+$ ground state, which is 0.3196 MeV below the *expected parity* $J = 1/2^-$ state. This is unmistakable evidence that there are low-lying intruders in this region.

B. The calculation

We present the results for the deformed oscillator model in Table VIII. This table contains both the input parameters and the results for the intrinsic state energies, and the energies of the $J = 0^+$ and $J = 2^+$ intruder states.

We first give Σ_x, Σ_y, Σ_z from which the frequencies ω_x, ω_y, ω_z, and ω_0 are obtained. This is sufficient to obtain the intrinsic state energies in units of $\hbar\omega_0$. Next the quantities needed to get the energies of the $J = 0^+$ and $J = 2^+$ states are shown. These are the expectation values $\langle J_x^2 \rangle$, $\langle J_y^2 \rangle$ and $\langle J_z^2 \rangle$ and the moment of inertia in units of $(\hbar\omega_0)^{-1}$. We then present the zero point energy ΔE_R in units of $(\hbar\omega_0)$. We then present $(\hbar\omega_0)$ using the formula $\hbar\omega_0 = 45\, A^{-1/3} - 25\, A^{-2/3}\, MeV$. It would be better to fit $\hbar\omega_0$ to experiment. However, since ^8Be is unstable one cannot measure the r.m.s. radius. There is no data available for ^{10}Be and for ^{12}C the error bars on r.m.s. are fairly large. At any rate, since we next present results for $E^*_{J=0}$ both in units of $(\hbar\omega_0)$ and in MeV, it is easy for the reader to obtain results for an $\hbar\omega_0$ of his/her choice. We lastly give the excitation energies of the $J = 2^+$ states.

Let us first discuss ^{12}C because the experimental situation here is most solid. The values of Σ_x, Σ_y and Σ_z for the ground state are 10, 6, 10. This implies $\omega_x = \omega_z < \omega_y$. This means that the y-axis is the symmetry axis and the nucleus will be oblate. The values of Σ_x, Σ_y and Σ_z for the $4p-4h$ intruder state are 6, 6, 18. Hence the z-axis will be the symmetry axis and the intrinsic state is prolate. We obtain the excitation energy of the $4p-4h$ $J = 0^+$ state to be $E^*_{J=0^+} = 6.55\, MeV$. The experimental value is 7.65 MeV. Considering the simplicity of this model the agreement is remarkable, and we must take the predictions of this model seriously, even if we do not fully understand why it works so well.

Rather than use the approximate formula $\hbar\omega_0 = \left(45\, A^{-1/3} - 25\, A^{-2/3}\right) MeV$ we can for a given nucleus fit the mean square charge radius, provided this quantity has been measured. This is not the case for ^8Be (unstable) or ^{10}Be, but for ^{12}C De Vries et al. [19] give three results due to different groups, $\langle r^2 \rangle = 2.472\,(15)$, $2.471\,(6)$ and $2.464\,(12)$ fm.

In our formulation the charge radius is given by

$$\langle r^2 \rangle_{ch} = \frac{\hbar^2}{Zm}\left(\frac{\Sigma_{\pi z}}{\hbar\omega_z} + \frac{\Sigma_{\pi x}}{\hbar\omega_x} + \frac{\Sigma_{\pi y}}{\hbar\omega_y}\right) \tag{17}$$

If we take $\langle r^2 \rangle^{1/2} = 2.47$ fm, we find $\hbar\omega_0 = 15.85$ MeV. This is larger than the value in Table VIII. We now find that the excitation energy of the $J = 0^+$ $4p-4h$ state is 6.97 MeV. This is closer to the experimental value of 7.654 MeV, than the value using the approximate formula for $\hbar\omega_0$ ($6.55 MeV$).

For ^{10}Be the values of Σ_x, Σ_y and Σ_z for the ground state are 7, 5, 9; for the $2p-2h$ intruder state they are 5, 5, 13. Thus the ground state band is triaxial but the intruder state has axial symmetry. We obtain $E^*_{J=0^+} = 6.36\, MeV$ in close agreement with the experimental result of 6.11 MeV.

We also include results for the axial symmetry approximation for the ground state of ^{10}Be. We replace the numbers 7, 5, 9 by 6, 6, 9. This might seem like a modest change. However, this is not the case. Indeed we find that the $2p-2h$ intruder state is 4.03 MeV below the axial ground state. This is due to a combination of reasons. First, the axial intrinsic ground state is 3.1 MeV above the triaxial intrinsic ground state. Secondly, we get a large zero point shift in the triaxial case because we get contributions from all three axes in the expression $\Delta E_R = \Delta E_x + \Delta E_y + \Delta E_z$. Again, if we had made the axial approximation for the $0p-0h$ state we would have reached the erroneous conclusion that the $2p-2h$ intruder state was the ground state. By correctly taking into account the triaxiality the situation gets reversed.

We now come to our main focus, the intruder states in ^8Be. We consider both the $2p-2h$ and the $4p-4h$ intruders. We find that the excitation energies are much higher than in ^{10}Be or ^{12}C. The $J = 0^+$ $2p-2h$ state is at 17.23 MeV and the $J = 0^+$ $4p-4h$ state is at 32.34 MeV in this calculation. We can understand this behaviour by considering the Nilsson diagram shown in Fig. 1. For ^{10}Be and ^{12}C we take nucleons from upward-going lines in the p shell and put them into a down-going line in the $s-d$ shell. The energy required to do this is much less for finite β than it is for $\beta = 0$, as can be easily seen from Fig. 1. For ^8Be, on the other hand we must take 2 nucleons from a down-going Nilsson line. This obviously costs much more energy. The figure and the corresponding argument make it quite convincing that the presence of low-lying intruder states in ^{10}Be and ^{12}C does not imply that there will be low-lying intruders in ^8Be.

VII. CONCLUSIONS

Because of the important implications to astrophysics of the ^8Be nucleus, we feel that Barker's suggestion [3,4] to worry about the presence of low-lying intruder states in this and neighboring nuclei is quite sensible. However, our

calculations do not support the presence of low-lying intruder states in ^8Be,i.e., of a $J = 2^+$ intruder at 9 MeV (which would also imply a $J = 0^+$ intruder at 6 MeV). Our lowest $J = 0^+$ intruder in the deformed oscillator model is above 17 MeV and the $J = 2^+$ above 19 MeV. These energies are lower than the 26 MeV gate mentioned by Warburton in the abstract of his 1986 work [2], but are sufficiently high so as not to seriously affect the alpha spectrum.

Our case was made more convincingly the fact that the same calculation does yield low-lying intruders in ^{10}Be and ^{12}C. In ^{12}C we are in close agreement with experiment (6.55 MeV vs. 7.654 MeV exp.). In ^{10}Be our calculated $J = 0^+$ state energy is very close to that of the first excited 0^+ state (6.36 MeV vs. 6.111 MeV exp.). However more experimental work will have to be done to determine if this is indeed an intruder state. Another possibility is that the 6.11 MeV state is the $J = 0^+$ member of an $L = 1, S = 1$ triplet with orbital symmetry [411] or [331].

Some questions remain. Why are the shell model energies higher than the deformed oscillator ones. It may be due to the truncated space used in the shell model calculations. If this is the case then this indicates a rather slow convergence. It would be of interest to try to enlarge the model space to test out this idea. It should be emphasized that in the $Q \cdot Q$ calculations the parameter χ was chosen carefully so that the energy of the first 2^+ state came out correctly. As we enlarge the model space we choose χ so that the fit to the 2^+state is maintained. This means that χ becomes smaller as the model space is increased.

We lastly express wonderment that the deformed oscillator model, with zero point energy corrections, seems to work so well in getting the intruder states at close to the right energies. In shell model calculations with realistic interactions it is very difficult to get the intruder states to come down low enough. This is because one starts with a spherical basis where for say ^{12}C the starting point energy for the $4p-4h$ state is $4\hbar\omega = 59.5 MeV$. One has to get the state down to 7.65 MeV and this is very difficult. It would be interesting to see whether this can be done with other realistic interactions suitably tailored for these type of calculations. In any case, the model space to do this must be enormous. However the deformed oscillator model almost effortlessly gets the state close to this energy. The Nilsson diagram in Fig. 1 explains in part this success but it would be nice to have a more quantitative understanding.

VIII. ACKNOWLEDGEMENTS

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Figure Captions

Figure 1.- Schematic Nilsson energies as a function of deformation.

Table I. $J = 0^+$ and 2^+ states in ^8Be for the interaction $-\chi Q \cdot Q$ with $\chi = 0.3365\ MeV/fm^4$ with up to $4\hbar\omega$ excitations allowed. The percentage of $0\hbar\omega$, $2\hbar\omega$ and $4\hbar\omega$ occupancies are given, as well as the $B(E2)(0_1^+ \rightarrow 2_i^+)$.

		(a) $J = 0^+$ $T = 0$ States		
$E_{exc}(MeV)$	0 $\hbar\omega$	2 $\hbar\omega$	4 $\hbar\omega$	
0.00	64.6	24.6	10.7	
11.37	83.4	10.9	5.7	
15.88	94.4	2.1	3.5	
17.86	94.3	2.5	3.2	
19.38	94.9	2.1	3.0	
26.23	28.5	50.9	20.6	
29.70	3.3	77.3	19.4	
32.08	0.0	86.1	13.9	
34.20	0.0	86.8	13.2	
35.93	13.8	70.7	15.4	
		(b) $J = 2^+$ $T = 0$ States		
$E_{exc}(MeV)$	0 $\hbar\omega$	2 $\hbar\omega$	4 $\hbar\omega$	$B(E2)_{0_1^+ \rightarrow 2_i^+}$ $(e^2 fm^4)$
3.04	66.3	23.8	9.9	65.3
11.37	83.4	10.9	5.7	0.0
13.59	86.2	8.9	4.9	0.0
15.88	94.4	2.1	3.5	0.0
15.95	87.5	8.3	4.2	0.0
17.86	94.3	2.5	3.2	0.0
19.39	94.9	2.1	3.0	0.0
27.15	28.5	51.4	20.2	15.7
30.22	0.0	79.3	20.7	0.0
31.71	1.0	80.1	18.9	1.6
32.09	0.0	86.1	13.9	0.0
33.87	0.1	83.3	16.6	0.0
34.20	0.0	86.8	13.2	0.0
35.71	10.7	75.0	14.3	0.0

Table II. Same as Table I but for the interaction $-\chi Q \cdot Q + xV_{s.o.}$ with $\chi = 0.3365\ MeV/fm^4$ and $x = 1$.

(a) $J = 0^+$ $T = 0$ States				
$E_{exc}(MeV)$	0 $\hbar\omega$	2 $\hbar\omega$	4 $\hbar\omega$	
0.0	65.1	24.0	10.9	
12.8	83.6	10.3	6.1	
16.4	89.7	6.0	4.3	
21.9	91.7	4.6	3.7	
26.4	69.3	21.3	9.4	
26.5	40.7	44.0	15.3	
29.9	3.4	77.4	19.2	
32.1	0.0	86.6	13.4	
37.3	0.0	85.6	14.3	
38.4	18.2	66.2	15.6	
(b) $J = 2^+$ $T = 0$ States				
$E_{exc}(MeV)$	0 $\hbar\omega$	2 $\hbar\omega$	4 $\hbar\omega$	$B(E2)_{0_1^+ \to 2_i^+}$ $(e^2 fm^4)$
3.1	66.7	23.3	10.1	63.4
10.2	85.8	8.8	5.4	0.4
13.2	88.2	7.2	4.6	0.9
16.2	91.9	4.2	3.9	0.0
17.7	86.4	8.9	4.7	0.2
19.6	88.3	7.4	4.3	0.0
21.6	84.8	10.3	4.9	0.1
22.2	91.0	5.1	3.8	0.0
27.5	27.8	53.1	19.1	14.5
30.9	0.9	78.0	21.0	0.0
31.9	1.1	80.2	18.7	1.6
32.4	0.0	86.2	13.8	0.0
34.3	0.2	85.7	14.0	0.0
34.6	1.2	83.8	15.1	0.1
35.2	11.4	74.0	14.6	0.1

Table III. Same as Table I but for the realistic (x, y) interaction with $x = 1$ and $y = 1$.

(a) $J = 0^+$ $T = 0$ States				
$E_{exc}(MeV)$	0 $\hbar\omega$	2 $\hbar\omega$	4 $\hbar\omega$	
0.0	62.2	26.2	11.6	
22.8	66.5	23.6	9.9	
28.7	6.5	71.0	22.5	
30.3	66.5	23.0	10.5	
35.3	67.5	22.4	10.1	
39.4	7.3	73.4	19.3	
43.5	56.3	31.4	12.3	
47.6	8.8	70.5	20.7	
49.5	2.3	76.7	21.6	
50.1	3.3	75.7	21.0	
(b) $J = 2^+$ $T = 0$ States				
$E_{exc}(MeV)$	0 $\hbar\omega$	2 $\hbar\omega$	4 $\hbar\omega$	$B(E2)_{0_1^+ \to 2_i^+}$ $(e^2 fm^4)$
5.4	62.2	26.6	11.1	31.1
19.5	70.0	20.4	9.6	0.0
21.5	69.5	20.2	10.3	0.1
26.2	69.7	20.5	9.8	0.4
30.4	70.2	20.9	8.9	0.0
31.0	56.7	30.9	12.6	1.7
33.7	13.5	65.7	20.8	3.7
35.1	71.3	19.7	9.0	0.0
38.2	67.7	22.4	9.8	0.0
41.6	9.0	68.8	22.2	1.3
45.0	1.0	79.7	19.3	0.1
45.9	2.9	77.9	19.2	2.4
46.3	3.2	76.7	20.1	1.3
47.3	0.3	79.5	20.2	0.0
48.4	1.5	79.8	18.6	0.0

Table IV. $J = 0^+$ and 2^+ states in ^{10}Be for the interaction $-\chi Q \cdot Q$ with $\chi = 0.3615\ MeV/fm^4$ with up to $2\hbar\omega$ excitations allowed. The percentage of $0\hbar\omega$ and $2\hbar\omega$ occupancies are given, as well as the $B(E2)(0_1^+ \rightarrow 2_i^+)$.

(a) $J = 0^+$ $T = 1$ States			
$E_{exc}(MeV)$	$0\ \hbar\omega$	$2\ \hbar\omega$	
0.0	81.8	18.2	
3.7	81.0	19.0	
7.3	93.6	6.4	
9.7	0.0	100.0	
12.1	92.9	7.1	
12.1	92.9	7.1	
13.9	93.1	6.9	
17.7	98.9	1.1	
22.1	0.0	100.0	
22.9	0.0	100.0	
(b) $J = 2^+$ $T = 1$ States			
$E_{exc}(MeV)$	$0\ \hbar\omega$	$2\ \hbar\omega$	$B(E2)_{0_1^+ \rightarrow 2_i^+}$ $(e^2 fm^4)$
2.2	81.3	18.7	5.0
3.4	83.4	16.6	47.2
3.7	81.0	19.0	0.0
7.3	93.6	6.4	0.0
9.2	82.9	17.1	0.0
10.9	91.9	8.1	0.0
11.9	0.0	100.0	0.0
12.1	92.9	7.1	0.0
12.1	92.9	7.1	0.0
12.1	92.9	7.1	0.0
13.9	93.1	6.9	0.2
13.9	93.1	6.9	0.0
13.9	93.1	6.9	0.0
17.7	98.9	1.1	0.0
22.1	0.0	100.0	0.0

Table V. Same as Table IV but for the interaction $-\chi Q \cdot Q + xV_{s.o.}$ with $\chi = 0.3615\ MeV/fm^4$ and $x = 1$.

(a) $J = 0^+$ $T = 1$ States			
$E_{exc}(MeV)$	$0\ \hbar\omega$	$2\ \hbar\omega$	
0.0	85.6	14.4	
8.0	80.8	19.2	
9.6	92.0	8.0	
11.4	0.0	100.0	
12.1	91.5	8.5	
16.4	90.6	9.4	
19.7	90.5	9.5	
23.1	88.7	11.3	
24.0	0.0	100.0	
26.1	0.0	100.0	
(b) $J = 2^+$ $T = 1$ States			
$E_{exc}(MeV)$	$0\ \hbar\omega$	$2\ \hbar\omega$	$B(E2)_{0_1^+ \to 2_i^+}$ $(e^2 fm^4)$
3.0	85.5	14.5	40.1
4.6	83.7	16.3	3.4
6.8	90.8	9.2	0.3
7.8	83.5	16.5	3.7
11.8	84.8	15.2	0.1
13.0	91.2	8.8	0.1
13.8	0.0	100.0	0.0
14.1	90.9	9.1	0.0
14.8	90.9	9.1	0.0
15.5	90.3	9.7	0.0
17.2	90.0	10.0	0.1
17.2	88.0	12.0	0.0
18.2	90.3	9.7	0.1
21.2	89.0	11.0	0.0
23.0	52.8	47.3	0.0

Table VI. Same as Table IV but for the realistic (x,y) interaction with $x=1$ and $y=1$.

(a) $J=0^+$ $T=1$ States		
$E_{exc}(MeV)$	$0\ \hbar\omega$	$2\ \hbar\omega$
0.0	73.3	26.7
8.7	74.4	25.6
12.0	74.7	25.3
21.1	76.5	23.5
23.7	77.5	22.5
31.0	49.3	50.7
31.5	25.4	74.6
34.5	5.8	94.2
37.6	0.6	99.4
39.7	74.1	25.9

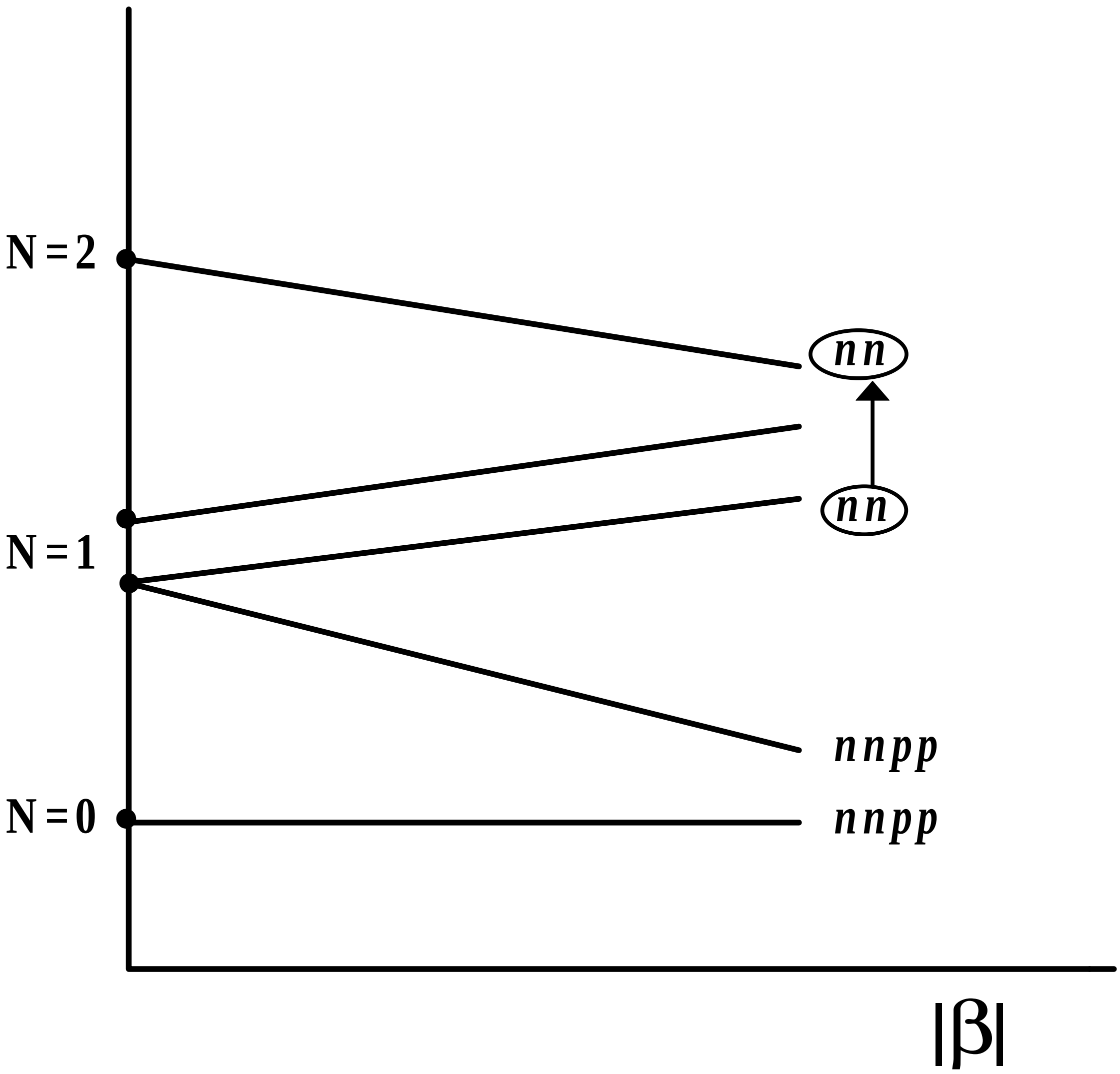
(b) $J=2^+$ $T=1$ States			
$E_{exc}(MeV)$	$0\ \hbar\omega$	$2\ \hbar\omega$	$B(E2)_{0_1^+\to 2_i^+}$ (e^2fm^4)
4.6	73.5	26.5	19.7
5.2	73.9	26.1	3.2
9.2	73.7	26.3	1.5
10.1	75.8	24.2	0.0
17.4	74.5	25.5	0.0
19.7	75.7	24.3	0.1
20.2	77.0	23.0	0.0
22.1	76.9	23.1	0.2
22.9	77.1	22.9	0.0
23.7	77.2	22.8	0.0
27.2	76.8	23.2	0.0
29.0	76.9	23.1	0.2
32.5	76.9	23.1	0.0
33.4	0.3	99.7	0.0
35.5	71.7	28.3	0.2

Table VII. Excitation energies (in MeV) of the first $J=0^+$ and 2^+ intruder states in ^8Be and ^{10}Be.

	$Q\cdot Q$	$Q\cdot Q\ +\ xV_{s.o.}$	(x,y)=(1,1)
		^8Be $J=0^+$ $T=0$	
$2\hbar\omega$	32.1	30.1	33.8
$4\hbar\omega$	26.5	26.5	28.7
		^8Be $J=2^+$ $T=0$	
$2\hbar\omega$	31.5	30.9	36.6
$4\hbar\omega$	27.5	27.5	33.7
		^{10}Be $J=0^+$ $T=1$	
$2\hbar\omega$	9.7	11.4	31.0
		^{10}Be $J=2^+$ $T=1$	
$2\hbar\omega$	11.9	13.8	33.6

Table VIII. Excitation energies of the first $J = 0^+$ and 2^+ intruder states in ^8Be, ^{10}Be, and ^{12}C in the deformed oscillator model.

		$\Sigma_x, \Sigma_y, \Sigma_z$	$\frac{\omega_x}{\omega_0}, \frac{\omega_y}{\omega_0}, \frac{\omega_z}{\omega_0}$	E_{int} $[\hbar\omega_0]$	$\left\langle J_x^2 \right\rangle, \left\langle J_y^2 \right\rangle, \left\langle J_z^2 \right\rangle$	$\mathcal{I}_x, \mathcal{I}_y, \mathcal{I}_z$ $\left[(\hbar\omega_0)^{-1}\right]$	ΔE_R $[\hbar\omega_0]$	$E^*_{J=0}$ $[\hbar\omega_0]$	$\hbar\omega_0$ [MeV]	$E^*_{J=0}$ [MeV]	$E^*_{J=2}$ [MeV]
^8Be	$0p-0h$	4,4,8	1.26,1.26,0.63	15.12	6,6,0	15.9,15.9,0	0.38		16.25		3.07
	$2p-2h$	4,4,10	1.36,1.36,0.54	16.29	10.5,10.5,0	21.4,21.4,0	0.49	1.06	16.25	17.23	19.51
	$4p-4h$	4,4,12	1.44,1.44,0.48	17.31	16,16,0	27.7,27.7,0	0.58	1.99	16.25	32.34	34.10
^{10}Be	$(0p-0h)_{triaxial}$	7,5,9	0.97,1.36,0.76	20.41	5.6,2.3,2.4	15.6,19.2,10.8	0.70		15.50		2.70
	$2p-2h$	5,5,13	1.38,1.38,0.53	20.63	14.4,14.4,0	28.2,28.2,0	0.51	0.41	15.50	6.36	8.01
	$(0p-0h)_{axial}$	6,6,9	1.14,1.14,0.76	20.61	3.75,3.75,0	17.0,17.0,0	0.22	0.67	15.50	10.39	13.12
^{12}C	$0p-0h$	10,6,10	0.84,1.41,0.84	25.30	5.3,0,5.3	16.1,0,16.1	0.33		14.89		2.77
	$4p-4h$	6,6,18	1.44,1.44,0.48	25.96	21,21,0	38.5,38.5,0	0.55	0.44	14.89	6.55	7.71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N = 0
nn
nn
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